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ABSTRACT

This book contains activities related to fractions and proportions. The activities span many grade levels as well as many levels of sophistication. Each activity is accompanied by a set of Teachers Notes that include a discussion of the mathematics topics, grade range, materials needed, mathematics of the activity, and implementation notes. All activities also include one or more handouts for use directly with students. (KHR)

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Classroom Activities

for Making Sense of Fractions, Ratios, and Proportions

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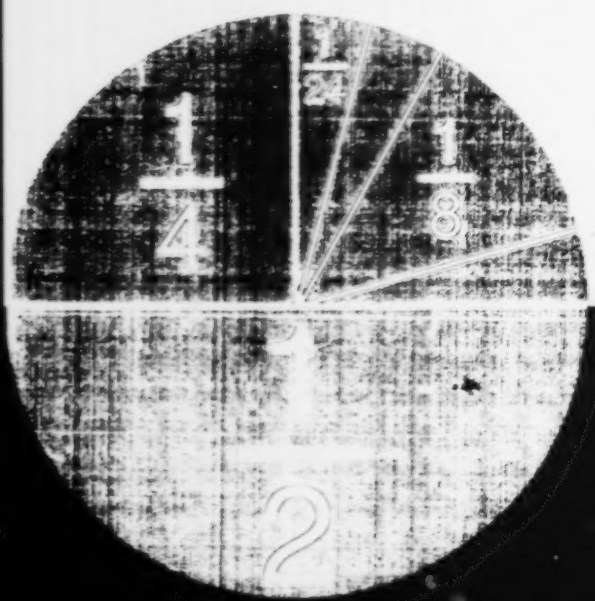
2002 Yearbook



National Council of Teachers of Mathematics

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Classroom Activities

for Making Sense of Fractions, Ratios, and Proportions

2002 Yearbook

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National Council of Teachers of Mathematics

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Table of Contents

1. **A Word to Teachers** 1

2. **Developing an Understanding of Equivalence Using Fair Sharing** 2
   JEFFREY C. SHIH
   University of Nevada—Las Vegas, Las Vegas, Nevada
   JOHN RATIU
   Loreto Street Elementary School, Los Angeles, California

3. **Finding One Half** 5
   RALPH CONNELLY
   Brock University, Saint Catherines, Ontario

4. **Fractions: The "Whole" Story** 7
   RALPH CONNELLY
   Brock University, Saint Catherines, Ontario

5. **Pizza** 9
   MARIA E. MAGONE
   University of Pittsburgh, Pittsburgh, Pennsylvania
   BARBARA M. MOSKAL
   Colorado School of Mines, Golden, Colorado
   SUZANNE LANE
   University of Pittsburgh, Pittsburgh, Pennsylvania

6. **Ordering Fractions** 11
   KATHLEEN CRAMER
   University of Minnesota—Twin Cities, Minneapolis, Minnesota

7. **Renaming It** 14
   SUSAN J. LAMON
   Marquette University, Milwaukee, Wisconsin

8. **Can You See It?** 17
   SUSAN J. LAMON
   Marquette University, Milwaukee, Wisconsin

9. **Using Your Percent Sense** 20
   BONNIE SCHAPPELLE
   San Diego State University, San Diego, California

10. **Fraction Area** 25
    MARIA E. MAGONE
    University of Pittsburgh, Pittsburgh, Pennsylvania
    BARBARA M. MOSKAL
    Colorado School of Mines, Golden, Colorado
    SUZANNE LANE
    University of Pittsburgh, Pittsburgh, Pennsylvania

11. **Special Sets of Number Lines** 29
    CLAIR HABERMAN
    North Saint Paul, Minnesota

12. **Nomograph for Adding Thirds and Fifths** 32
    CLAIR HABERMAN
    North Saint Paul, Minnesota

13. Addition Nomograph 34
CLAIR HABERMAN
North Saint Paul, Minnesota

14. Numbers Need Not Apply 36
ESTHER M. H. BILLINGS
Grand Valley State University, Allendale, Michigan

15. Cocoa 38
ESTHER M. H. BILLINGS
Grand Valley State University, Allendale, Michigan

16. John's School 41
MARIA E. MAGONE
University of Pittsburgh, Pittsburgh, Pennsylvania
BARBARA M. MOSKAL
Colorado School of Mines, Golden, Colorado
SUZANNNE LANE
University of Pittsburgh, Pittsburgh, Pennsylvania

17. Make a New Puzzle 43
LISA CLEMENT
San Diego State University, San Diego, California

18. Ratio Conceptualization Using the Human Body 46
GAIL M. GALLITANO
West Chester University of Pennsylvania, West Chester, Pennsylvania

19. Proportional Reasoning as a Bridge to Algebra: Lemonade Mix Activity 49
FRANCINE CABRAL ROY
University of Rhode Island, Kingston, Rhode Island

20. Centimeters and Inches: A Different Look at Proportional Reasoning 52
SUZANNE LEVIN WEINBERG
Temple University, Philadelphia, Pennsylvania

21. Conversion of Measurements: Unit Analysis 56
ELIZABETH B. APPELBAUM
Blue Valley School District, Overland Park, Kansas

1

A Word to Teachers

With the publication of the 2002 Yearbook, *Making Sense of Fractions, Ratios, and Proportions*, we begin a new tradition—the publication of a Classroom Companion that contains activities that illustrate some of the ideas in the yearbook and activities that go beyond the content of the yearbook itself. The intent of the Classroom Companion series is to bring each yearbook to life in the classroom by providing sample activities that can help students understand and use important mathematical ideas.

This first Classroom Companion, *Activities for Making Sense of Fractions, Ratios, and Proportions*, contains activities related to proportional reasoning. As noted in *Principles and Standards for School Mathematics*, proportional reasoning is a crucial element of middle grades mathematics. It is important as both a foundation of many applications of mathematics to real life and an essential element of more sophisticated mathematics (e.g., algebra).

The activities span many grades, as well as many levels of sophistication. Remember, however, that these activities are only samples; there is no attempt to be comprehensive of all the ways that students might be supported in developing clear mathematical understanding of ratio and proportions. We encourage you to use these samples as a launching pad for expanding your thinking about other ways to help students learn.

Each activity is accompanied by a set of Teachers Notes. These notes typically include a discussion of the mathematics topics, the grade range, materials needed, the mathematics of the activity, and implementation notes. All the activities (except the first one) also include one or more handouts for use directly with students. You can certainly find many ways to adapt or expand these activities to fit the specific needs of your students.

We hope that you will find these activities useful, both directly with students and in expanding your thinking about the teaching of proportional reasoning. We encourage you to read both the chapters in the yearbook itself, so that you have the benefit of the thinking of those authors, and the new Navigations books that have been written to complement *Principles and Standards for School Mathematics*.

Editorial Panel:

George W. Bright, *General Editor*
Bonnie Litwiller, *Issue Editor*
David Duncan
Judy Sowder
Teresa Yazujian

2

Developing an Understanding of Equivalence Using Fair Sharing: Teaching Notes

Jeffrey C. Shih

John Ratiu

GRADE RANGE: K–2

MATHEMATICAL TOPICS: equivalence, fair sharing, partitive division

MATERIALS: Access to fraction manipulatives such as fraction circles and bars should be provided for those students who choose to use them.

Problems

1. Show in two or three ways how 3 people can share 15 pizzas fairly.

2. Show in two or three ways how 3 people can share 1 pizza fairly.

Discussion of the Mathematics

Fair sharing offers children an opportunity to build on their informal understanding of sharing to learn about fractions. The context of fair sharing is flexible enough to allow a variety of partitioning strategies as students develop an understanding of the relationship between the number of sharers and the number of things being shared. By varying the numbers and the contexts in fair-sharing problems, a wide range of mathematics can be addressed. With these two problems, we wanted our students to see the similarity between their solution strategies for sharing 15 pizzas and 1 pizza.

Because of the flexibility of solution strategies that fair-sharing problems allow, the partitioning strategies that students use to solve the problems offer opportunities to develop an understanding of equivalence. Discussion about equivalence can occur whenever two students partition a quantity in equivalent amounts. For example, one student might solve the first problem and arrive at the answer 5 wholes, whereas another student might say that the answer is 15 thirds. This discussion is particularly important in that it challenges students' beliefs that mathematics problems have only one answer.

Implementation

After reading the problems together as a class, allow the students ample time to solve the problems using a variety of partitioning strategies. As you watch students work, ask them to explain their strategies and push them for clarification and more mathematical detail (Carpenter et al. 1999). Then in a whole-group discussion, have several students share their solution strategies so the class can discuss the strategies. For us, the requirements were that the students always had to explain their solution strategies and had to try to solve the problems in multiple ways. If they finished, they challenged themselves by changing the numbers in the problem or by writing new problems to solve.

Questions to Ask Students

How much pizza does each person get? Are the pieces the same size? Why is that important? Can we plan in advance how many slices we make? How can we cut the pizza so that everyone gets the biggest slice possible?

Students' Typical Responses

For the first problem, two seven-year-old students, Samantha and Jennifer, were selected to explain their solutions to the class. Jennifer directly modeled the problem, drawing 15 pizzas and dealing pizzas to each of the three people (see fig. 2.1).

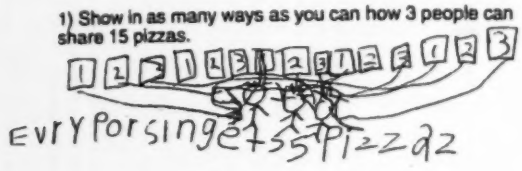

Fig. 2.1. Jennifer's solution

Jennifer's answer to the problem was "Five." After Jennifer answered questions from her classmates about her strategy, her teacher asked the class how many of them solved the problem using a strategy like Jennifer and if anybody had any more questions for her. The class seemed satisfied with her solution strategy.

Next to share was Samantha, who chose to divide each of her pizzas into three pieces, giving one piece of each pizza to each person (see fig. 2.2).

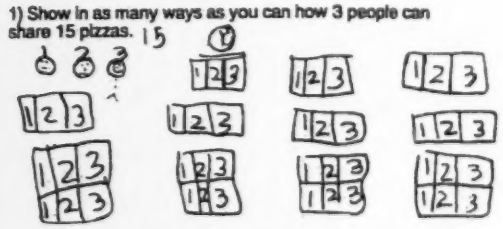

Fig. 2.2. Samantha's solution

Samantha's answer was "Fifteen." Again, the teacher asked the class if they had used a strategy similar to Samantha to solve the problem and if there were any questions for Samantha. Many students nodded their heads in response to the follow-up question of whether they thought Samantha's solution strategy was correct. However, when asked what the correct answer was to the problem, part of the class responded, "Five!" whereas others answered, "Fifteen!" The students also believed there could be only one correct answer to the problem. With the day ending, Mr. Ratiu decided to wait until the next day to pose the following problem.

Follow-up Problem

> Yesterday in class, Jennifer's answer was 5 pizzas for each person. Samantha's answer was 15 slices of pizza. Who was right? Please explain why.

The students attempted to solve the problem using "Samantha's way" and "Jennifer's way." They engaged in a rich discussion about what the "five" and "fifteen" referred to. "How can they be the same?" was heard often. This type of student-initiated questioning was more evidence that the children were developing an understanding of equivalence. Figure 2.3 shows a sample of some of the students' solutions:

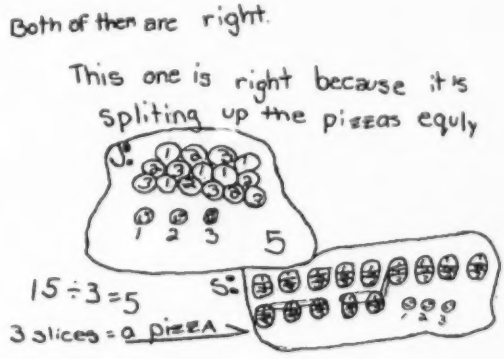

Fig. 2.3. A comparison of the solutions

Both of them are right. Because they did it in different ways, Samantha did slices and Jennifer did whole pizzas. But they both gave everyone 5 pizzas. But they gave everyone 5 pizzas in <u>different</u> ways."

"I think both of them are right because all you have to do is just cut Jennifer's pizzas into pieces so person number 1 number 2 and number 3 gets the same answer. And that is 15 slices (Samantha) or 5 pizzas (Jennifer)."

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Additional Information

Sequencing fair sharing after division worked quite well from a mathematical standpoint and from the perspective of the types of solution strategies our students were using. Students were familiar with dealing discrete amounts to solve division problems, and the transition toward partitioning wholes into fractions followed smoothly given the students' familiarity with the various food contexts. We also wanted to give our students experiences in fairly sharing a wide variety of shapes and contexts. For example, pizzas and cupcakes provide round contexts, whereas licorice sticks and candy bars are rectangular. By building on children's developing understanding of division and their keen understanding of sharing and food, we found that the students' background knowledge supported meaning-making and allowed them the opportunity to focus on the mathematical concepts being presented.

Reference

Carpenter, Thomas P., Elizabeth Fennema, Megan L. Franke, Linda Levi, and Susan B. Empson. *Children's Mathematics: Cognitively Guided Insturction.* Portsmouth, N.H.: Heinemann, 1999.

3

Finding One-Half: Teaching Notes

Ralph Connelly

GRADE RANGE: 1–2

MATHEMATICAL TOPIC: exploring one half of a region

Discussion of the Mathematics, with Answers and Solutions

The real world can present challenges to a child's fraction understanding. For example, if a child is sharing a cookie with a friend and breaks it in two parts to do so, we say it is polite to give the friend "the bigger half." So children are led to believe that anytime something is divided into two pieces, it is cut "in half." This activity helps children understand that one-half means one of two equal parts. It also makes them aware that one-half of a region can be shaded even if there are more than two pieces. This helps develop informally ideas such as 2 of 4 equal pieces shaded is the same area as 1 of 2 equal pieces shaded. Answer: The figures on the activity sheet that are not half-shaded are 4, 11, 12, and 14.

Implementation

This activity can be used effectively as an introductory activity to begin a discussion of what one-half of a region means or as a culminating activity to assess children's understanding of one-half of a region.

Questions to Ask Students

Use the cookie idea to ask children about times when they have taken one-half of something. After the activity, talk about what one-half of a chocolate bar with 4 equal pieces would be.

Students' Typical Responses

If a child selects items 1, 2, 3, 4, 7, 10, 11, 12, 13, and 14, this might suggest that the child believes you can have one-half only if there are two parts and one of them is shaded. The child who selects the same figures except for 4, 11, 12, and 14 would appear to recognize that the "halves" have to be equal, but he or she does not seem aware that, for example, if 3 of 6 equal parts of a region are shaded, then this is also one-half of the region. Some children may see that 8 and 15 are half-shaded but not be able to see that 5, 6, and 9 are as well. Some might not select 3 because "it doesn't go from corner to corner."

Further Reading

Watanabe, Tad. "Ben's Understanding of One-Half." *Teaching Children Mathematics* 2 (April 1996): 460–64.

Name _____ Date _____

Finding One-Half

Circle the figures you think are one-half shaded.

1.

2.

3.

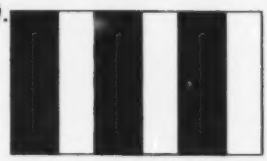
4.

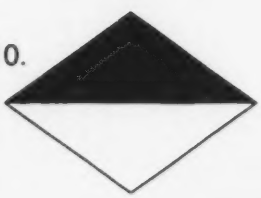
5.

6.

7.

8.

9. 

10. 

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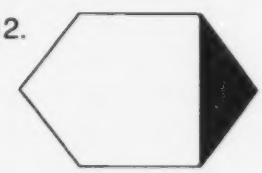
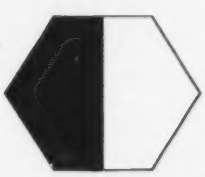
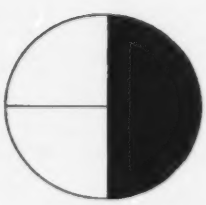
12. 

13. 

14.

15. 

4

Fractions–the "Whole" Story: Teaching Notes

Ralph Connelly

GRADE RANGE: 3–5

MATHEMATICAL TOPIC: fractions as part of a whole

MATERIALS: pattern blocks (yellow hexagons, red trapezoids, blue rhombii, green triangles)

Discussion of the Mathematics, with Answers and Solutions

Children need experiences with whole regions other than circular ones in order to develop fraction understanding more fully. For example, displaying ◺ and asking "what fraction is this?" would almost assure an answer of "1/4" without consideration of any other possibility. However, displaying △ and asking the same question might yield answers like "it's a whole thing" or "it depends."

Answers

1. 2
2. 1/2
3. 3
4. 1/3
5. 6
6. 1/6
7. 1/4
8. 1/6

Implementation

Students can complete the activity individually (for assessment purposes) or in pairs or groups.

Questions to Ask Students

You might ask the question above as an introduction to the activity. After the activity, you might go back to the same question to see if students now recognize various possibilities—that is, the answer depends on what we select to represent the whole.

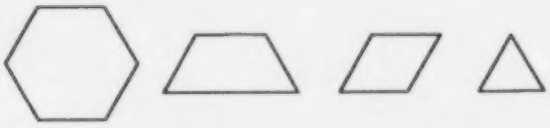
Students' Typical Responses

Students are usually successful with questions like 1, 3, and 5, when the answer is a whole number, but they might need some guidance in writing answers to the other questions as a fraction. For questions 7 and 8, students might respond as though one hexagon is the whole rather than the double hexagon, and they may need to be encouraged to look again at what figure represents the whole region.

Name _____ Date _____

Fractions–the Whole Story

Use these pattern blocks



to answer the following questions:

1. If = 1, what is ?

2. If = 1, what is ?

3. If = 1, what is ?

4. If = 1, what is ?

5. If = 1, what is ?

6. If = 1, what is ?

7. If = 1, what is ?

8. If = 1, what is ?

5

Pizza: Teaching Notes

Maria E. Magone

Barbara M. Moskal

Suzanne Lane

GRADE RANGE: 2–5

MATHEMATICAL TOPIC: representing fractions

Discussion of the Mathematics

Students must connect a pictorial representation and a symbolic representation of a fraction.

Answers:

2. 1/3;

4. 1 (1/3).

Questions to Ask Students

Does each picture match the answer you wrote? Explain how your picture shows your answer. What is the "unit"? (What represents 1?)

Students' Typical Responses

Common incorrect responses include the following:

- Dividing the pizza into triangular slices similar to the cut of round pizzas
- In Problems 3 and 4, using the set of pizzas rather than an individual pizza as the unit of measure
- Writing the number of slices rather than the fraction of the pizza as the answer

Credits

This task was codeveloped, as part of the Quantitative Understanding: Amplifying Student Achievement and Reasoning Project Cognitive Assessment Instrument (QCAI), by Suzanne Lane, Edward A. Silver, Robert D. Ankenmann, Jinfa Cai, Connie Finseth, Mai Liu, Maria E. Magone, David Meel, Barbara M. Moskal, Carol S. Parke, Clem A. Stone, Ning Wang, and Yuehua Zhu.

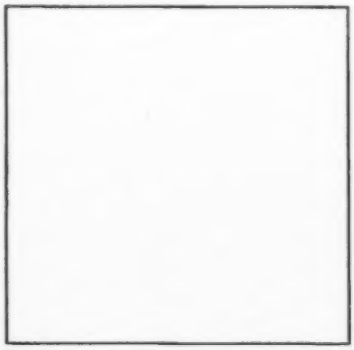
NOTE: This activity accompanies chapter 13, "Making Explicit What Students Know about Representing Fractions," of the 2002 Yearbook, *Making Sense of Fractions, Rations, and Proportions*. The chapter describes how this activity was used with students. This description will be helpful to you in thinking about how to use this activity.

Name _____ Date _____

Pizza

Luis, Elena, and Leslie plan to share 1 large, square pizza. Each person will get an equal amount.

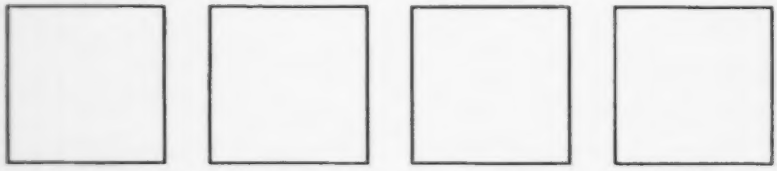
1. Show on the picture how much pizza Elena will get.



2. What fraction of the pizza will Elena get?

Maria, Carlos, and Terry want to share 4 medium, square pizzas. Each person will get an equal amount.

3. Show on the picture how much pizza Carlos will get.



4. How many pizzas will Carlos get?

6

Ordering Fractions: Teaching Notes

Kathleen Cramer

GRADE RANGE: 4–6

MATHEMATICAL TOPIC: ordering fractions

MATERIALS: fraction circles or other manipulatives (as needed by students)

Discussion of the Mathematics

Three types of fraction sets are discussed in this lesson: (*a*) fractions close to 0, 1/2, and 1, (*b*) fractions with the same numerator, and (*c*) fractions with the "same number of missing parts from the whole." Students should try to use their mental images to order these fractions instead of changing fractions to equivalent fractions with like denominators.

Implementation

If students have not had sufficient experience with manipulative models, you might want to conduct a whole-class discussion similar to the one outlined below. If students have difficulty with any of the fraction sets, ask them to model the fractions with fraction circles or some other manipulative.

- Write a group of fractions on the overhead that have the same numerator (e.g., 3/5, 3/9, and so on). Ask students to order the fractions and then to describe their strategies. Help students see that since the numerators are all the same, the larger denominators are associated with smaller fractions. Connecting this idea to fraction circles might help clarify this issue.
- Write a group of fractions on the overhead that have large denominators and small numerators (e.g., 1/9, 2/12, and so on). Ask students to order the fractions and then to describe their strategies. Help students see that these fractions are all close to zero.
- Write a group of fractions on the overhead whose numerators are 1 less than their denominators (e.g. 11/12, 3/4, and so on). Ask students to order the fractions and then to describe their strategies. Help students see that these fractions are all "missing" one piece to make a whole. A small missing piece corresponds to a large fraction. Connecting this idea to fraction circles might help clarify this issue.
- Write one of these pairs of fractions on the overhead: 9/12 and 2/6 or 4/12 and 3/4. Ask students to select the larger fraction. Help students see that in these cases, one fraction is greater than 1/2 whereas one is less than 1/2.

Questions to Ask Students

After students complete the activity page, challenge them to apply their number sense to problems like these: Is the answer to 5/6 + 11/12 greater or less than 1? Greater or less than 2? Greater or less than 1 1/2? (Repeat for 4/7 + 6/11 and 1/5 + 3/4.) Students' answers to the addition problems may sound like the following:

- Since both fractions are a little less than 1, the sum is close to 2 but not as much as 2.

- Since both fractions are just a bit more than 1/2, the sum will be greater than 1, but less than 1 1/2.
- 3/4 is greater than 1/2; 1/5 is less than1/4. The sum is greater than 1/2 but less than 1, since you would need 1/4 more to equal 1.

Students' Typical Responses

When students compare fractions, they often make connections to concrete models. When comparing fractions with like numerators (e.g., 3/4 and 3/5), students might say, "Fourths are bigger than fifths; three of the bigger pieces are more than three of the smaller pieces." When comparing 11/12 and 3/4, students might say, "11/12 is the bigger fraction. Both fractions are one away from the whole; with twelfths, the gap is smaller, since 1/12 is smaller than 1/4." When comparing 9/12 and 2/6, students might say, "9/12 is bigger, since it is more than 1/2 and 2/6 is less than 1/2." Often students realize that a fraction is greater than 1/2 when the numerator is more than half the denominator.

Credit

Cramer, Kathleen, Merlyn Behr, Richard Lesh, and Thomas Post. *The Rational Number Project: Fraction Lessons for the Middle Grades–Level 2.* Dubuque, Iowa: Kendall/Hunt Publishing Co., 1997.

Name _____ Date _____

Ordering Fractions

Picturé these fractions in your mind. Using your knowledge of fraction sense, order the fractions in each set from smallest to largest.

1. $\frac{99}{100}$ $\frac{6}{7}$ $\frac{15}{16}$

___ ___ ___

2. $\frac{4}{10}$ $\frac{4}{47}$ $\frac{4}{8}$

___ ___ ___

3. $\frac{9}{16}$ $\frac{1}{14}$ $\frac{8}{9}$ $\frac{5}{12}$

___ ___ ___ ___

4. $\frac{12}{15}$ $\frac{5}{8}$ $\frac{97}{100}$

___ ___ ___

5. $\frac{19}{20}$ $\frac{2}{22}$ $\frac{6}{14}$

___ ___ ___

6. $\frac{11}{15}$ $\frac{11}{24}$ $\frac{11}{19}$

___ ___ ___

7. Which fractions are close to 0? _____

8. Which fractions are close to 1/2? _____

9.. Which fractions are close to 1? _____

7

Rename It: Teaching Notes

Susan J. Lamon

GRADE RANGE: 4–7

MATHEMATICAL TOPIC: equivalent fractions

MATERIALS: Use the accompanying sheet containing nonstandard units of area: a rec, a railroad track, a long, a stick, a block, a ministick, and a minirec.

Discussion of the Mathematics

These activities encourage children to generate many names for the same area by mentally chunking that area into different-sized pieces instead of by "multiplying the numerator and the denominator by the same whole number." This process, known as *unitizing*, uses measurement principles and reasoning rather than rule-based procedures. First with simple quantities and then using part-whole comparisons, students write equivalent names for a given area, indicating both the number and the size of the pieces being used. For example,

$$\frac{1 \text{ minirec}}{4 \text{ minirecs}} = \frac{\frac{1}{6} \text{ block}}{\frac{2}{3} \text{ block}} = \frac{\frac{1}{4} \text{ railroad track}}{1 \text{ railroad track}}$$

Implementation

Pieces should not be cut out and used as manipulatives. Students should be encouraged to think and to visualize.

Questions to Ask Students

Who can name that area in a different way? How can you be sure that these names are equivalent? What happens when you rename with larger pieces? What happens when you rename with smaller pieces?

Students' Typical Responses

When they get to problem 3, students often remark that 1 stick + 1 minirec looks like a flag. At that point, you may define a new unit of area called a *flag* and generate new questions: 1 rec = ___ flags, 1/2 flag = ___ railroad tracks, and so on.

Complex fractions should be encouraged because they highlight important relationships; for example, 1 minirec to 4 minirecs is the same relative amount of area as ___ of a railroad track is to a whole railroad track. This helps students to be comfortable with fractions whose components are not always "nice" numbers.

Additional Information

You may increase the difficulty of this activity by changing the dimensions of rec. For example, if rec is 14 units by 4 units and the other pieces remain the same, more mixed numbers will be needed.

Credits

Lamon, Susan J. *Teaching Fractions for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers.* Mahwah, N.J.: Lawrence Erlbaum Associates, 1999.

NOTE This activity accompanies chapter 9, "Part-Whole Comparisons with Unitizing," in the 2002 NCTM Yearbook, *Making Sense of Fractions, Ratios, and Proportions.* Please refer to this chapter for more information on unitizing.

Name _____ Date _____

Rename It

Find as many different names as you can for each area

1. 1 rec = 3 railroad tracks =

2. 5 railroad tracks =

3. 1 rec – 1 1/2 blocks =

4. 1 stick + 1 minirec =

5. 1 rec – 2 minirecs =

6. 3 ministicks + 2 longs =

7. 1 block – 1 railroad track =

8. 1 rec – 3 ministicks =

9. $\frac{\text{1 ministick}}{\text{3 ministicks}} = \frac{\text{1/2 minirec}}{\text{1 1/2 minirecs}} =$

10. $\frac{\text{1 minirec}}{\text{4 minirecs}} =$

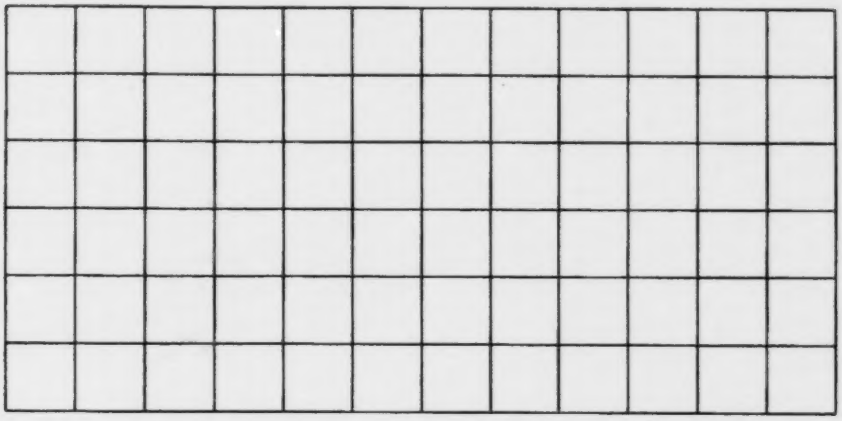
11. $\frac{\text{2 railroad tracks}}{\text{3 railroad tracks}} =$

12. $\frac{\text{1 1/2 sticks}}{\text{1 stick}} =$

13. $\frac{\text{3 sticks}}{\text{2 sticks}} =$

14. $\frac{\text{1 stick}}{\text{5 sticks}} =$

15. $\frac{\text{1/2 rec}}{\text{2 recs}} =$

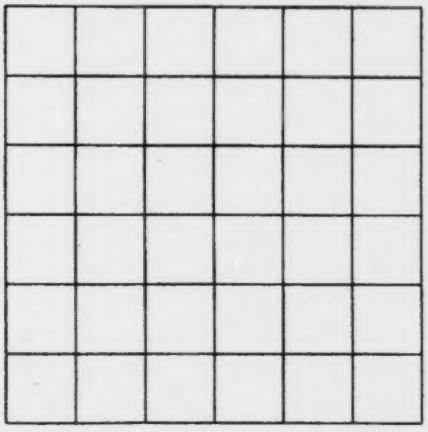


Rec

Railroad Track

Stick

Long



Block

Ministick

Minirec

8

Can You See It?: Teaching Notes

Susan J. Lamon

GRADE RANGE: 4–7

MATHEMATICAL TOPIC: reasoning with part-whole fractions

Discussion of the Mathematics

These activities are designed to follow Rename It. They use unitizing (thinking in terms of different-sized chunks) to facilitate reasoning with fractions. Without mentioning the word *multiplication* and without any rules, students are able to perform fraction multiplication. For example, the first problem is equivalent to computing 2/3 • 18 = 12.

Answers

1. $\frac{4 \text{ pair}}{9 \text{ pair}}$, 8 cookies

2. $\frac{5\ (1\frac{1}{2}\text{-packs})}{12\ (1\frac{1}{2}\text{-packs})}$, $7\frac{1}{2}$ cookies

3. $\frac{5\ (3\text{-packs})}{6\ (3\text{-packs})}$, 15 cookies

4. $\frac{14\ (\frac{1}{2}\text{ cookies})}{36\ (\frac{1}{2}\text{ cookies})} = 7$ cookies

5. $\frac{3\ (4\frac{1}{2}\text{-packs})}{4\ (4\frac{1}{2}\text{-packs})} = 13\frac{1}{2}$ cookies

6. $\frac{6\ (4\text{-packs})}{9\ (4\text{-packs})}$

7. $\frac{24 \text{ cookies}}{36 \text{ cookies}}$

8. $\frac{2 \text{ dozen}}{3 \text{ dozen}}$

9. $\frac{8\ (3\text{-packs})}{12\ (3\text{-packs})}$

10. $\frac{8\ (3\text{-packs})}{12\ (3\text{-packs})}$

11. $\frac{3\ (8\text{-packs})}{4\frac{1}{2}\ (8\text{-packs})}$

12. $\frac{1\frac{1}{3}\ (9\text{-packs})}{4\ (9\text{-packs})}$

CREDITS:

Lamon, Susan J. *Teaching Fractions for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers.* Mahwah, N.J.: Lawrence Erlbaum Associates, 1999.

NOTE: This activity accompanies chapter 9, "Part-Whole Comparisons with Unitizing," in the 2002 NCTM yearbook, *Making Sense of Fractions, Ratios, and Proportions.* Please refer to this chapter for more information on unitizing.

Name _____ Date _____

Can You See It?

Here are some lemon cookies.

Look at the cookies to find the answer to each question.

Example. Can you see thirds? How many cookies will you eat if you eat 2/3 of the lemon cookies?12 cookies

$$\frac{2\text{ (6-packs)}}{3\text{ (6-packs)}}\text{, 12 cookies}$$

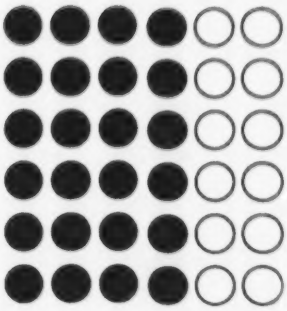
1. Can you see ninths? How many lemon cookies will you eat if you eat 4/9 of the cookies?

2. Can you see twelfths? How many lemon cookies will you eat if you eat 5/12 of the cookies?

3. Can you see sixths? How many lemon cookies will you eat if you eat 5/6 of the cookies?

4. Can you see thirty-sixths? How many lemon cookies will you eat if you eat 14/36 of the cookies?

5. Can you see fourths? How many lemon cookies will you eat if you eat 3/4 of the cookies?



Here are some chocolate and vanilla cookies.

Look at the cookies to find the answer to each question.

Example. If we put these cookies in 6 packs, how would you name the part of the cookies that are chocolate?

$$\frac{\text{4 (6-packs)}}{\text{6 (6-packs)}}$$

6. If we put the cookies in 4-packs, what part of the cookies are chocolate?

7. If we wrap each cookie separately, what part of the cookies are chocolate?

8. If we pack the cookies in dozens, what part of the cookies are chocolate?

9. If we pack the cookies in 3-packs, what part of the cookies are chocolate?

10. The vanilla cookies fill 4 packages. Use packages of the same size to name the chocolate cookies.

11. The vanilla cookies fill 1 1/2 packages. Use packages of the same size to name the chocolate cookies.

12. The chocolate cookies fill 2 2/3 packages. Use packages of the same size to name the vanilla cookies.

9

Using Your Percent Sense: Teaching Notes

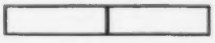
Bonnie Schappelle

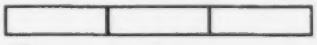
GRADE RANGE: 4–7

MATHEMATICAL TOPIC: Find 100% when given a part of, or when given more than, 100%

Discussion of the Mathematics, with Answers and Solutions

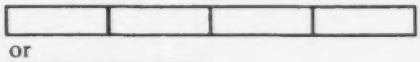
Students may not have been taught procedures for solving problems like these, so they will need to reason about the meaning of percent and the relationships of the parts to the whole. These activities can also be used to assess their understanding of percent. Possible solutions follow:

1. (a.)  50%;

(b.) 

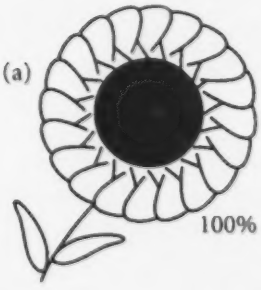
or

75%

(c) 

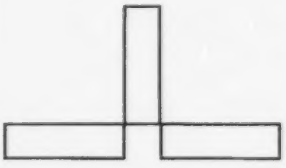
or

100%

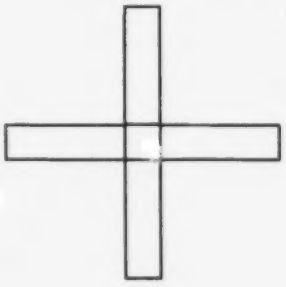
2. (a) 

100%

(b) 5%

3. (a.) 

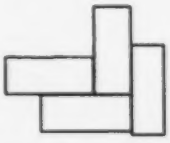
25 % of larger figure shaded;

(b.) 

100%

4.

50% of that "rectangle" shaded

5. 

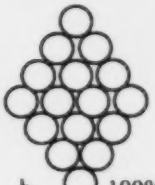
100% of the "smaller figure" shaded

6. a. 20%;

b. 100% shaded

7. 100%

8. a. 10% of the smaller figure



b. 100% of the smaller figure shaded

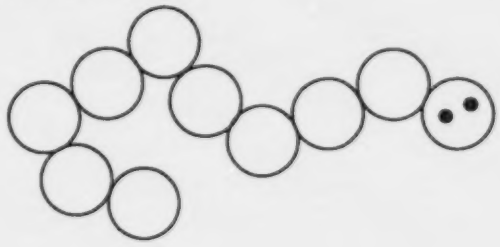
9. 100% of the smaller figure

Implementation

Students can be encouraged to create such problems for their peers to solve. They could be asked to create some that are easy and some that are more difficult to solve; to explain what makes a given problem easy or difficult to solve, they will need to think even more deeply about the relationships of parts to wholes. Although most of these problems are not set in real-world situations, such settings could be included—and the children could embed their own problems in real-world settings of interest to them. An example: "The following drawing shows how much cake is left after 40% of the people at the party have had their fair shares of this cake. If a total of 20 people came to the party, how should you cut the remaining cake so that those who have not yet had their portions receive their fair shares? Complete the drawing to show how the whole cake might have looked before anyone was served cake."

(Example of remaining cake cut "so that those who have not yet had their portions receive their fair shares.")

(Example of whole cake before anyone was served cake.)



Questions to Ask Students

Can you think of situations in which 150% is a reasonable amount?

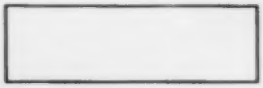
CREDIT:

Mathematics Resource Project. *Ratio, Proportion, and Scaling*. Preliminary ed. Eugene, Ore.: University of Oregon, 1977.

Name _____ Date _____

Using Your Percent Sense

1. This is 25% of a candy bar.



(a) Draw 50% of the candy bar.

(b) Draw 75% of the candy bar.

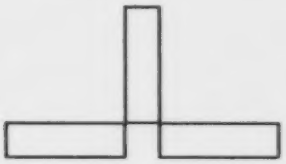
(c) Draw 100% of the candy bar.

2. These are 30% of the petals from a flower.

(a) Show how the flower might have looked with 100% of its petals.

(b) What percentage of the flower is one petal?

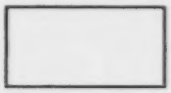
3. This is 75% of a figure.



(a) Shade 25% of that figure.

(b) Draw 100% of the figure.

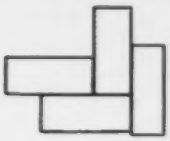
4. This is 150% of a rectangle.



(a) Shade 50% of that rectangle.

(b) Explain why the unshaded part is 100% of that rectangle.

5. This is 200% of a smaller figure.



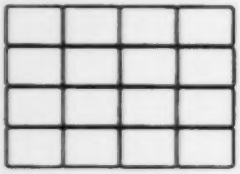
Shade the smaller figure.

6. This is 120% of a smaller figure.

a. Draw 20% of the smaller figure.

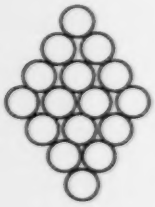
b. Shade 100% of the smaller figure.

7. This is 800% of a smaller design.



Draw 100% of the smaller design.

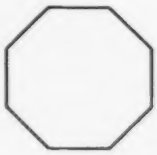
8. This is 160% of a smaller figure.



a. Draw 10% of the smaller figure.

b. Shade 100% of the smaller figure.

9. This is 160% of a smaller figure.



Shade the smaller figure.

Fraction Area: Teaching Notes

Maria E. Magone

Barbara M. Moskal

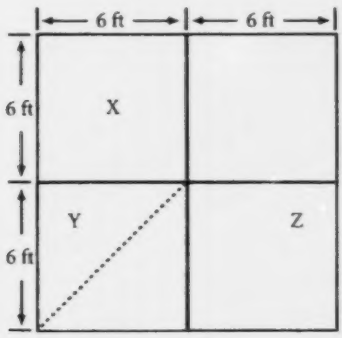
Suzanne Lane

GRADE RANGE: 5–8

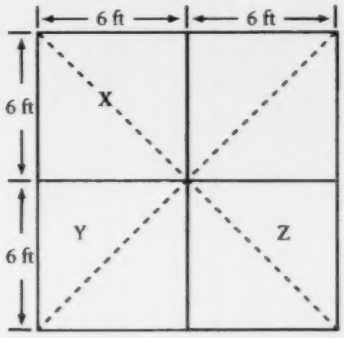
MATHEMATICAL TOPICS: fractions, measurement, geometry

Discussion of the Mathematics

Solutions can be obtained by subdividing larger regions into smaller, congruent regions. See the diagrams below.



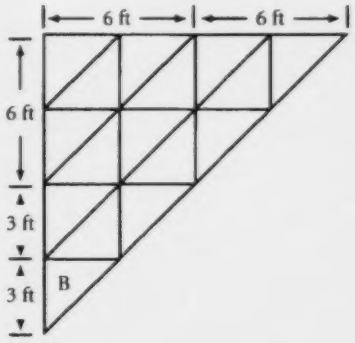
1. Divide the region into four squares as shown above. Answer: $X = 1/4$.



2. Divide the region into eight triangles as shown by the dotted segments above. Answer: $Y = 1/8$

3. Answer: $Z = 5/8$.

The answers to Questions 5 and 6 can be found in several alternative ways, as shown below.



5. Divide the region into 16 small triangles. Answer: $B = 1/16$.

6. Count small triangles for region A. Answer: $A = 15/16$.

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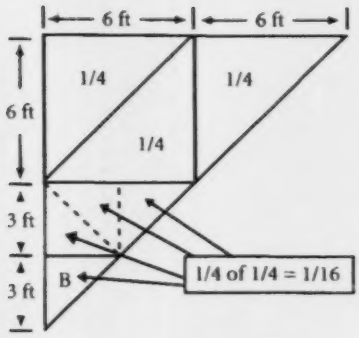


5. Divide the region into 4 large triangles. Divide the triangle containing "*B*" into small triangles. Each small triangle will be 1/4 of 1/4 = 1/16. Answer: $B = 1/16$.

6. Answer: $A = 1 - 1/16 = 15/16$.

Questions to Ask Student:

What is the relationship between region *X* and region *Y*? How can you use what you know about region *X* and region *Y* to find the fractional part of the whole for region *Z*? How can you use what you know about region *B* to find the fractional part of the whole for region A?

Students' Typical Responses

Typical errors include the following:

• An answer of "2 1/2" for Problem 3. The student counted the number of squares (6 × 6 squares) and expresses the answer using this smaller square as the unit of measure.

• The student recognizes that region *Y* is a smaller fractional part than region *X* and that region *Z* is a larger fractional part than region *X* but is unable to determine the numerical relationship.

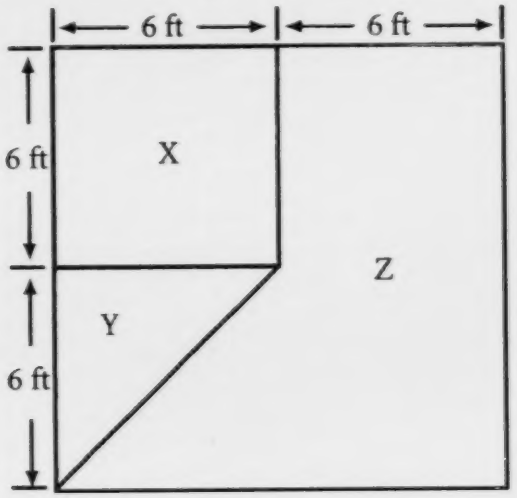
Credit

See credit for Chapter 5, "Pizza," on page 10.

Name _____ Date _____

Fraction Area

The large square below has been divided into three regions.



1. What fraction of the area of the large square is covered by region X?

2. What fraction of the area of the large square is covered by region Y?

3. What fraction of the area of the large square is covered by region Z?

4. Explain or show how you found your answers.

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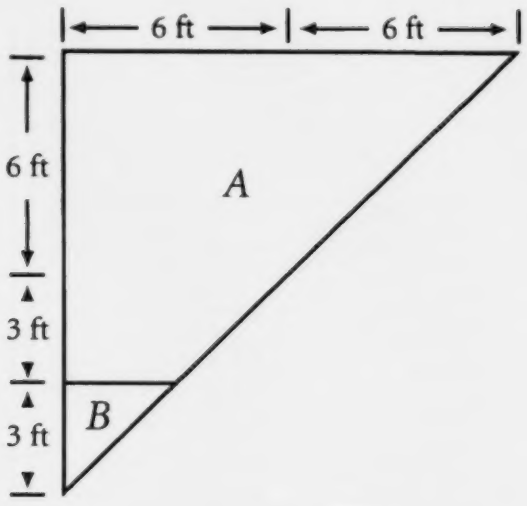
The large isosceles triangle below is divided into two regions.



5. What fraction of the area of the large triangle is covered by region *B*?

6. What fraction of the area of the large triangle is covered by region *A*?

7. Explain or show how you found your answers.

11

Special Sets of Number Lines: Teaching Notes

Clair Haberman

GRADE RANGE: 6–9

MATHEMATICAL TOPICS: adding and subtracting fractions

MATERIALS: very sharp pencils, rulers or straightedges

Discussion of the Mathematics, with Answers and Solutions

Before the advent of handheld calculators and computers, nomographs (sets of number lines) were sometimes used to perform specialized computations. The use of this nomograph can reinforce understandings of calculations with halves, which are assumed to be familiar to students.

There are two important characteristics of addition nomographs. First, the vertical distance between lines *A* and *S* is the same as between lines *S* and *B*. Second, the spaces between two numbers on line *S* is half of that between those numbers on line *A* or *B*. Graph paper is recommended for constructing nomographs. A search of the Internet, using *nomograph*, yielded hundreds of sites of varying degrees of relevance.

Implementation

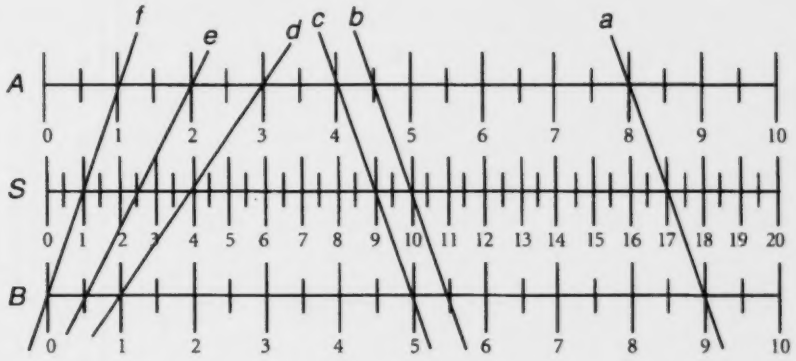
Students might work independently or in pairs. Students may confuse the printed numbers with the points and draw segments through the numbers rather than through the points on the number lines. If students have difficulty, overhead transparencies can be used for demonstration.

When students have completed the exercises, discuss answers for problem 8. Either "Addition" or "It adds *A* and *B*" is appropriate. The purpose here is to make explicit the property $A + B = S$. Problems 9 and 10 involve the relationship between addition and subtraction. As an extension, pose this question: "Select a point on line *S* and a point on line *A*. Draw a segment through them so that it intersects ine *B*. Express the relationship between the numbers on lines *S*, *A*, and *B*." By extending the number lines to the left of zero, the nomograph can be used for work with negative numbers.

Name _____ Date _____

Special Sets of Number Lines

Six segments are drawn on the number lines below.



Complete the chart to summarize the information about these six segments.

Segment	Line A	Line B	Line S
a	8	9	17
b			
c			
d			
e			
f			

1. Draw a segment joining 6 on line A with 7 on line B. Where does that segment cross line S?

2. Draw a segment joining 5 1/2 on line A with 6 on line B. Where does that segment cross line S?

3. Draw a segment joining 6 1/2 on line A with 8 1/2 on line B. Where does that segment cross line S?

4. Draw a segment joining 3 1/2 on line A with 1 1/2 on line B. Where does that segment cross line S?

5. If the lines were longer, where would a segment joining 31 on line A with 47 on line B cross line S?

6. If the lines were longer, where would a segment joining 46 1/2 on line A with 53 on line B cross line S?

7. If the lines were longer, where would a segment joining 51 1/2 on line A with 78 1/2 on line B cross line S?

8. Draw a segment through 9 1/2 on line B and 18 on line S so that it intersects line A. Where does it intersect line A? 18 – 9 1/2 = _____.

9. List some of the special properties of a nomograph.

10. Write an equation that indicates the relationship between the numbers on lines A, B, and S.

12

A Nomograph for Adding Thirds and Fifths: Teaching Notes

Clair Haberman

GRADE RANGE: 6–9

MATHEMATICAL TOPICS: adding and subtracting mixed numbers

MATERIALS: very sharp pencils, rulers or straightedges

Discussion of the Mathematics

As with the previous activity, nomographs provide a chance for students to explore addition and subtraction of fractions and mixed numbers.

Implementation

Be sure that students are connecting the points on lines A and B rather than connecting the fractions. You may need to help students create a process for finding differences. As students finish, you might challenge them to create a nomograph for halves and fifths or for fourths and fifths.

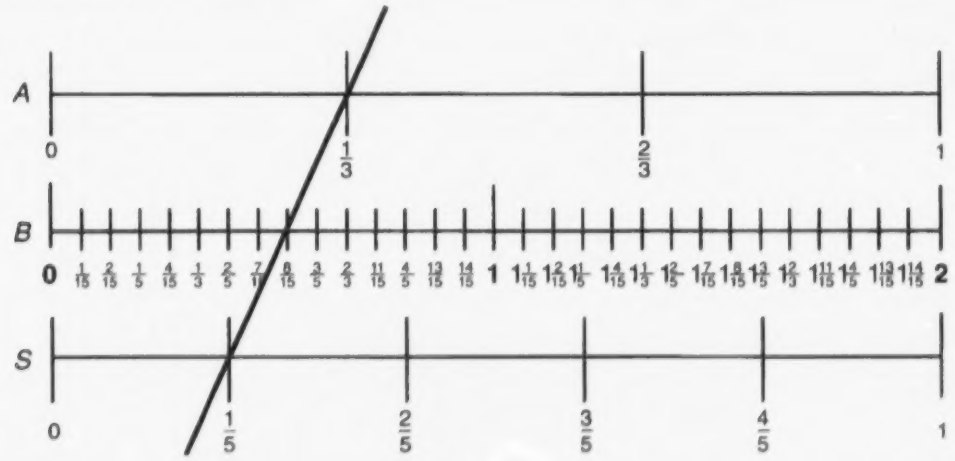
Questions to Ask Students

Ask students if their answers are the same as they would get from using a standard algorithm for adding fractions. Ask students whether they prefer using nomographs or using a standard algorithm.

Name _____ Date _____

A Nomograph for Adding Thirds and Fifths

To find the sum of 1/3 and 1/5, connect the points for 1/3 on line A and 1/5 for line B. The segment crosses line S at the point labeled 8/15.



Use the nomograph above to find these sums:

1. 2/3 + 1

2. 1 + 3/5

3. 1/3 + 2/5

4. 2/3 + 1/5

5. 1/3 + 3/5

6. 2/3 + 4/5

7. You know that 22 + 13 = 35. Use one of the answers above to find 22 1/3 + 13 3/5.

Without extending lines A, B, or S in the nomograph above, find each of the following sums.

8. 7 + 4 2/3

9. 9 1/3 + 4 2/5

10. 17 2/3 + 8 1/5

11. 31 2/3 + 20 4/5

12. Explain how you would use the nomograph to find the difference of two numbers.

Use the nomograph to find each difference:

13. 1 7/15 – 4/5

14. 1 4/15 – 2/3

15. 3 4/15 – 1 2/3

16. 8 1/15 – 3 2/5

13

An Addition Nomograph: Teaching Notes

Clair Haberman

GRADE RANGE: 6–9

MATHEMATICAL TOPICS: adding and subtracting mixed numbers

MATERIALS: very sharp pencils, rulers or straightedges

Discussion of the Mathematics

The key part of this activity is labeling the number lines in the nomograph. The labeling of line A in thirds and line B in fourths should be pretty straight- forward. Students might profit from a discussion of why the labeling of line S is twelfths (the lowest common multiple of 3 and 4).

Implementation

Students may need some help in understanding when a computation can be completed and when it cannot. The two major reasons for not being able to compute are (*a*) one of the lines is not long enough and (*b*) the segment drawn intersects line A or line B in between two labeled points. In the first case, students can imagine extending the lines. In the second case, students have to think about how the lines would have to be relabeled; this is a more difficult thing to imagine.

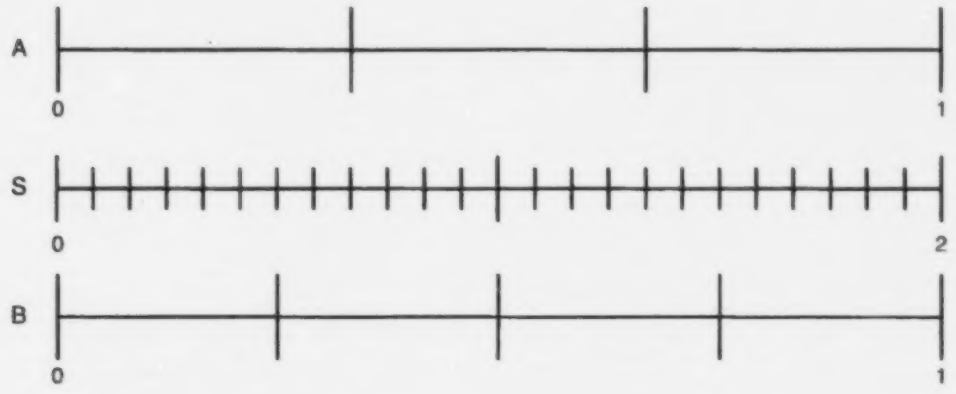
Questions to Ask Students

"What is an appropriate label for the intersection of line A and the segment drawn in exercise 10?" "Depending on their progress in the study of fractions, students may need hints with calculating $5/12 - 1/4 = (5 - 3)/12 = 2/12 = 1/6$. "How could number lines A and B be labeled so that answers for all possible subtractions could be obtained?" Students could construct a nomograph with lines A, B, and S labeled in twelfths, with appropriate reducing, and then determine if all possible additions and subtractions with twelfths could be performed. This will provide an illustration of the concept of closure.

Name _____ Date _____

An Addition Nomograph

1. Below is an unlabeled nomograph. What sets of fractions can be added using this nomograph?



2. Label the points on lines A, B, and S.

Use the nomograph to find each sum or difference:

3. 2/3 + 1/2

4. 4 2/3 + 7/12

5. 11/12 – 2/3

6. 3 11/12 – 1 2/3

7. 1 5/12 – 2/3

8. 9 5/12 – 6 2/3

9. Can you find 1 7/12 – 1/4? If yes, what is the difference? If no, explain why not.

10. Can you find 5/12 – 1/4? If yes, what is the difference? If no, explain why not.

14

Numbers Need Not Apply: Teaching Notes

Esther M. H. Billings

GRADE RANGE: 5–8

MATHEMATICAL TOPICS: proportions, ratios, multiplicative reasoning, proportion sense

Discussion of the Mathematics, with Answers and Solutions

These problems help cultivate students' proportion sense, that is, the ability to reason about quantities and the various relationships that quantities share with other quantities in proportional situations. Students often focus on the numerical quantities in a proportion problem and try to apply some known strategy to these quantities without really understanding why or if their strategy is appropriate for the situation. One way to help students understand proportional relationships is to provide "numberless" problems that encourage direct reasoning about the relationship between variables, how one variable affects another, and what these relationships indicate about the proportional situation. Most students are fairly successful in correctly identifying the answers for these problems. However, they may have trouble identifying the pertinent variables that directly affect the proportional relationship.

Answers

1. Catherine. Since Catherine arrives at the end destination first, she is riding faster because she covers the same distance as Rachel in a shorter time period. (Of course, this answer assumes that each girl rides the same distance and that neither took a short cut.)
2. 36-inch string. As the length of a piano string becomes shorter, its frequency of vibration (the speed at which the string moves back and forth) becomes faster. We could also say that the longer the piano string is, the slower its frequency of vibration. Choose the longer string, since it vibrates more slowly.
3. This problem provides a nice opportunity to talk about assumptions and the role of different variables. One possible answer is to assume that the walls they are building are the same thickness or depth (e.g., 2 blocks deep). In this case, since they use the same number of identical blocks, the only variables that vary are the height and length of the wall. Then, if Maria's wall is taller, Ann's wall is longer, since the volume of blocks used to build the wall is conserved. Another answer is to conclude that more information is needed, since the problem does not explicitly state that the walls are the same thickness (depth). Consequently, Maria's wall might be taller, but we can't say for sure that Ann's wall will be longer (in length) or thicker (in depth). For example, if each girl uses 24 blocks, Maria's wall could be 4 blocks high, 2 blocks thick, and 3 blocks long, and Ann's, 3 blocks high, 4 blocks thick, and 2 blocks long, making it shorter (in height and length) but thicker (in depth) than Maria's. (Ann's wall could also be 1 block high, 4 blocks thick, and 6 blocks long, making it thicker in depth, shorter in height, and longer in length.)

Credit

Billings, Esther M. H. "Qualitative Based Reasoning of Preservice Elementary School Teachers in Proportional Situations." (Ph. D. diss., Northern Illinois University, 1998). *Dissertation Abstracts International* 59 *no.* 09A (1998): 3383.

Name _____ Date _____

Numbers Need Not Apply

1. Catherine and Rachel like to ride their bikes along a bike trail. Today they started riding at the same time at the beginning of the trail. Each rode continuously at a constant speed (making no stops) to the end of the trail. Rachel took more time than Catherine to reach the end of the trail. Which girl was biking faster? Explain your answer.

2. The frequency of vibrations of a piano string increases as the length decreases. Which piano string would vibrate more slowly, a 36-inch string or a 24-inch string? Explain your answer.

3. Maria and Ann each have the same number of identical wooden (cube) blocks. Each girl is told to build a wall, using all of the blocks, so that there are no "holes" in the wall. Maria's wall is taller than Ann's. What can we say about Ann's wall? Explain your answer.

15

Cocoa: Teaching Notes

Esther M. H. Billings

GRADE RANGE: 5–8

MATHEMATICAL TOPICS: proportion, ratio, multiplicative reasoning, proportion sense

Discussion of the Mathematics, with Answers and Solutions

These problems help cultivate students' proportion sense, that is, the ability to reason about quantities and the various relationships that quantities share with other quantities in proportional situations. Since no scale is given for the volume of cocoa or concentration of cocoa mix to water in each thermos, encourage students to think generally about the scenarios, especially extreme cases (a very large amount of hot cocoa versus a minute amount of hot cocoa; very "thick" hot cocoa versus very weak and diluted hot cocoa) and the relative relationships between thermoses.

Answers

1. Thermos A. Adding mix to A makes it even stronger-tasting and adding water to B further dilutes it.

2. Thermos B. Though the initial taste is the same, the volumes differ. Adding cocoa mix to a mixture of lesser volume will have greater affect on the overall taste, so B now contains the cocoa with the stronger chocolate taste.

3. Thermos B. Since both thermoses contain the same volume there is more cocoa mix per unit of water in B, since it is initially stronger-tasting. Each thermos of cocoa becomes stronger-tasting. The relationship between thermoses remains the same, since B was initially stronger-tasting.

4. Indeterminate, since we don't know how much stronger B is than A. The cocoa becomes stronger-tasting in A and weaker-tasting in B after alterations are made. However, we can not conclude if the concentrations of cocoa have altered enough to change the relationship between thermoses. Depending on the initial ratios of scoops of cocoa mix to cups of water, different outcomes are possible. For example, if A starts with 1 scoop of mix and 2 cups of water and B with 9 scoops of mix and 2 cups of water, B still contains stronger-tasting cocoa after the alterations. But if A initially contains 4 scoops of mix and 2 cups of water and B contains 5 scoops of mix and 2 cups of water, making the alterations would make A contain the stronger-tasting cocoa.

An informal argument also works. (Reasoning informally may also help provide a starting place to choose appropriate numerical examples to show the indeterminacy of the situation.) If the cocoa in B is very "thick" and chocolatey, adding one cup of water may only change the consistency slightly. The cocoa in A may be so weak that you can hardly taste a chocolate flavor so adding one scoop of mix may only slightly alter the flavor and consistency. Consequently, the cocoa in B would still be stronger-tasting. However, the two thermoses may also contain cocoa that tastes almost the same with cocoa in B only slightly stronger-tasting. Making the alterations could cause a change in taste so that A now contains hot cocoa with a stronger chocolate taste.

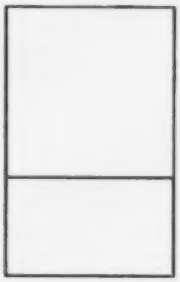
Credit

Billings, Esther M. H. "Qualitative Based Reasoning of Preservice Elementary School Teachers in Proportional Situations." (Ph. D. diss., Northern Illinois University, 1998). *Dissertation Abstracts International* 59, no. 09A (1998): 3383.

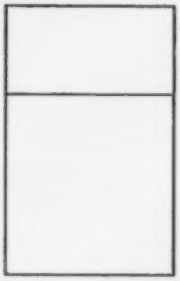
Name _____ Date _____

Cocoa

1. Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

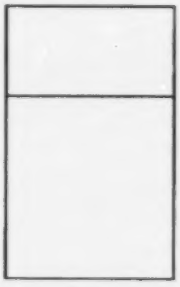


Thermos A

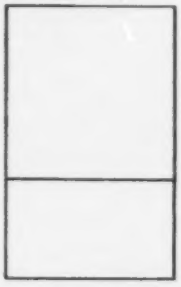


Thermos B

2. Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

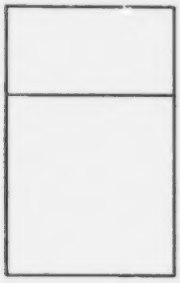


Thermos A

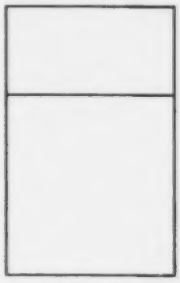


Thermos B

3. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

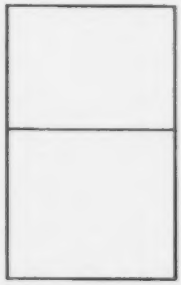


Thermos A

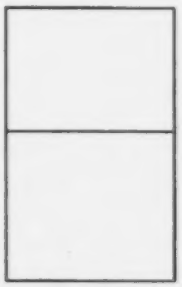


Thermos B

4. Thermos B. contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B. which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.



Thermos A



Thermos B

16

John's School: Teaching Notes

Maria E. Magone

Barbara M. Moskal

Suzanne Lane

GRADE RANGE: 6–8

MATHEMATICAL TOPICS: ratio, proportion, measurement

Discussion of the Mathematics

Students are asked to demonstrate an understanding of a proportional relationship by finding the missing dimension of one of the figures.

Answers

1. 45 short sticks
3. 100 short sticks

Questions to Ask Students

What is the relationship between short and long sticks in each drawing? How can you use this relationship to solve the problem?

Students' Typical Response

The ratio of the short stick to the long stick for problem 1 is 20:60 or 1:3, and since the factor (3) is a whole number, students are likely to be successful in solving this problem. The ratio for problem 2 is 30:75 = 50:125 = 1:2.5. Since the factor (2.5) is not a whole number, some students will have considerably more difficulty solving this problem.

Students may incorrectly use an additive approach for these problems. For problem 1, they might reason that 20 plus 40 is 60, so 15 plus 40 is 55. For problem 2 they might reason that 50 plus 75 is 125, so 40 plus 75 is 115; or that 30 plus 45 is 75, so 40 plus 45 is 85. If they try either of these strategies, you have a good opportunity to ask them about the other pair of given measurements as a means of challenging their thinking.

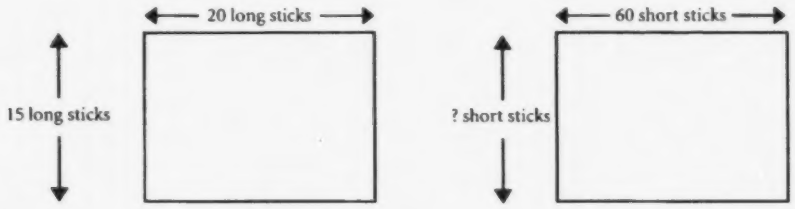
Credit

See credits for the notes to Chapter 5, "Pizza" on page 10.

Name _____ Date _____

John's School

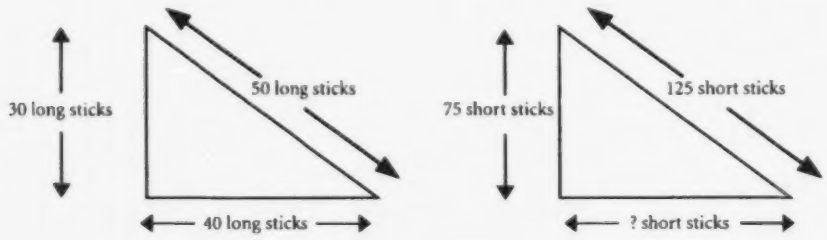
John used short and long sticks to measure his mathematics classroom. He drew these pictures:



1. What is the width of the room in short sticks?

2. Explain how you found your answer.

John used a different set of short and long sticks to measure his school's garden. He drew these pictures:



3. What is the length of the base of the garden measured in short sticks?

4. Explain how you found your answer.

17

Make a New Puzzle: Teaching Notes

Lisa Clement

GRADE RANGE: 5–8

MATHEMATICAL TOPICS: similarity, ratio, proportional reasoning, problem solving

MATERIALS: rulers with metric measures, scissors, unlined paper, protractors (optional)

Discussion of the Mathematics

The goal of this task is to have students determine that similarity involves comparing lengths multiplicatively rather than additively. Understanding that a multiplicative relationship is needed to describe corresponding lengths in similar figures is not obvious to all students. This idea can be quite difficult conceptually, particularly when the scale factor is something other than a whole number. The scale factors were specifically chosen to be nonwhole numbers (7/4, 8/5, and 2/3), so that students have to focus on a multiplicative approach for solving the problems.

Implementation

If the prior mathematical experiences of your students are not adequate, you may want to begin by having the students create a new puzzle with Rule 1a: The segment that measures 4 cm on the original should measure 8 cm on the new version. This is an easier problem than the other problems, since the scale factor is 2, a whole number.

Students' Typical Responses

Students will often erroneously use an additive approach for these tasks. For example, for Rule 1, students will often add 3 centimeters to every length to try to make a new piece of the same shape. After they make their pieces and try to fit them together to make a larger version of the original puzzle, however, they realize that the pieces do not fit together and must consider other approaches. Students will sometimes suggest an alternative and appropriate method by recognizing that 7 is 75% more than 4, so that adding 75% of each length to the original length will get the new length. Once students have described their methods, groups can try each method to determine which approach is the best. Students may also offer the following appropriate approaches:

> Ratio approach: For Rule 1, the ratio of the length in the new puzzle to the corresponding length in the original is 1 3/4.
>
> Scale-factor approach: For Rule 1, each length in the new puzzle is 1 3/4 times the corresponding length in original puzzle.

Questions to Ask Students

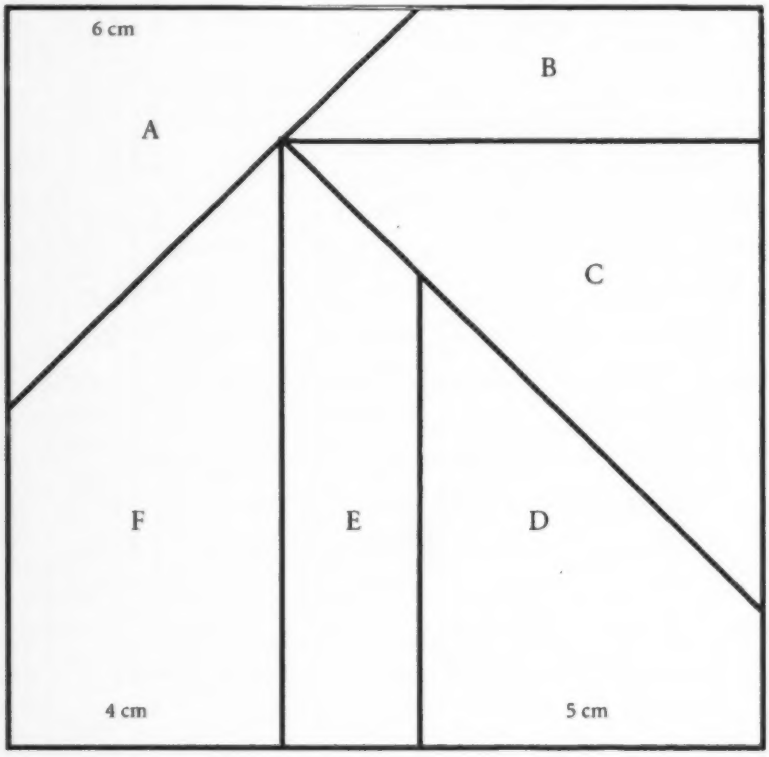
The richness of the task comes from having students grapple with the additive and multiplicative approaches. After the students have worked on creating the new square using Rule 1, you may ask them to come to the board or overhead to explain the strategies they used. Did you end up with a new square with every approach? Which approaches did not seem to work? Which approaches did work? How do you know they did or did not work? How could we describe the differences between the approaches that did work and the approaches that did not work?

If students do not suggest one of the approaches, you may want to suggest it yourself by stating, "I had a student last year who solved the problem this way..." (using the approach that they have not mentioned). If you put forth the approach as a student's and not your own, the students can debate the relative merits of it without directly accepting it simply because you (the teacher) suggested it. What if you know the length in the original puzzle and the length in the new puzzle; how could you find the ratio of the lengths? What if you know the ratio between the new lengths and the corresponding lengths in the original; how could you determine the lengths for the pieces in the new puzzle?

Name _____ Date _____

Make a New Puzzle

The pieces below may be cut out and then reassembled to make the square.



1. Make a new set of all six pieces so that Rule 1 is satisfied. Work in groups of three. Each group member selects two pieces to create and cut out. When each person is finished, you should be able to put the new pieces together to make a square.

   Rule 1: The segment that measures 4 cm on the original should measure 7 cm on the new version.

2. For each of Rules 2 and 3, make a sketch of all six pieces and indicate the measurements needed to get the new puzzle.

   Rule 2: The 5 cm segment on the original should measure 8 cm on the new version.

   Rule 3: The 6 cm segment on the original should measure 4 cm on the new version.

3. Write a set of instructions for enlarging and reducing the puzzle pieces. Describe any method that did not work, and explain why it did not work.

18

Ratio Conceptualization Using the Human Body: Teaching Notes

Gail M. Gallitano

GRADE RANGE: 4–9

MATHEMATICAL TOPICS: ratios, proportion, similarity of ratios of body parts

MATERIALS: measuring tapes (if available; otherwise, string and rulers or meter sticks), calculators

Discussion of the Mathematics, with Answers and Solutions

The underlying concept is ratio—the comparison of two amounts. Measurements of body parts can be used to generate interesting ratios that seem to match most people. Answers (approximate ratios): 1 for arm span to height, 1 for forearm to foot, 2 for wrist to perimeter of base of thumb, 2 for neck to wrist, 2 for waist to neck, 6 for height to foot. These whole numbers are really ratios with a denominator of 1. For example, the ratio of neck to wrist is really 2:1 or 2/1; that is, 1 time around the neck is the same as 2 times around the wrist. It is important in debriefing students' solutions to keep their attention focused on this ratio idea.

Implementation

Groups may measure each student or only one volunteer. The ratios match best for "average" students, so you need to be alert if unusually large students are measured. Measuring just one ("average") student in the group may ease the task of determining the whole-number, approximate ratios.

Questions to Ask Students

Have the students find other ratios using their measurements. For example: wrist/waist = 1/4; perimeter of base of thumb/waist = 1/8; height/forearm = 6/1.

Additional Information

The golden ratio is $(1 + \sqrt{5})/2$, and is usually called ϕ (phi). It is often approximated by 10/6 or 16/10. The number ϕ pops up in many unexpected places, especially in nature, art, and music. For example, the quantity ϕ occurs often in the structures of both the ancient Greeks and the ancient Egyptians. The Greek Parthenon contains many golden rectangles. The golden rectangle is considered the rectangular shape most pleasing to the eye. The ratio of the longer side to the shorter side is ϕ. Many famous artists have used the golden rectangle in their paintings, and architects have used it in their design of buildings. Much of the construction of the Egyptian pyramids was of triangular forms based on ratios involving ϕ. In studying the proportions of the human body, one will find the ratio frequently. In the human being, for example, the distance from the top of one's head to their waistline/the distance between their two shoulders = ϕ. Also, the distance from the top of one's head to their chin/the width of their forehead = ϕ, that is, approximately 1.6 (West et al. 1982). Students may be interested in taking these measurements and finding the ratio ϕ.

Credit

West, Beverly H., Ellen N. Griesbach, Jerry D. Taylor, and Louise T. Taylor. *Prentice Hall Encyclopedia of Mathematics.* Englewood Cliffs, N.J.: Prentice Hall, 1982.

Name _____ Date _____

Ratio Conceptualization Using the Human Body

1. Have students work in groups to take and record the following measurements of a volunteer group member. Make each measurement to the nearest centimeter (cm).

Measurements	
height _____ cm	arm span _____ cm
forearm _____ cm	perimeter of base of thumb _____ cm
length of foot _____ cm	waist _____ cm
wrist _____ cm	neck _____ cm

2. Compute the ratios for your group. For the decimal ratio, use a calculator and round to the nearest hundredth.

		Fraction		Decimal Ratio
$\frac{\text{arm span}}{\text{height}}$	=	$\frac{\text{cm}}{\text{cm}}$	=	_____
$\frac{\text{wrist}}{\text{perimeter of base of thumb}}$	=	$\frac{\text{cm}}{\text{cm}}$	=	_____
$\frac{\text{waist}}{\text{neck}}$	=	$\frac{\text{cm}}{\text{cm}}$	=	_____
$\frac{\text{forearm}}{\text{foot}}$	=	$\frac{\text{cm}}{\text{cm}}$	=	_____
$\frac{\text{neck}}{\text{wrist}}$	=	$\frac{\text{cm}}{\text{cm}}$	=	_____
$\frac{\text{height}}{\text{foot}}$	=	$\frac{\text{cm}}{\text{cm}}$	=	_____

3. Record the decimal ratios for all groups.

Ratio	Group 1	Group 2	Group 3	Group 4	Group 5
$\frac{\text{arm span}}{\text{height}}$	_____	_____	_____	_____	_____
$\frac{\text{forearm}}{\text{foot}}$	_____	_____	_____	_____	_____
$\frac{\text{wrist}}{\text{perimeter of base of thumb}}$	_____	_____	_____	_____	_____
$\frac{\text{neck}}{\text{wrist}}$	_____	_____	_____	_____	_____
$\frac{\text{waist}}{\text{neck}}$	_____	_____	_____	_____	_____
$\frac{\text{height}}{\text{foot}}$	_____	_____	_____	_____	_____

4. For each ratio identify the whole number that best approximates the decimal data you recorded. Write it as a whole number and as a ratio in the form_____:_____.

Ratio	Whole Number Approximation	Ratio Format
$\frac{\text{arm span}}{\text{height}}$	_____	_____ : _____
$\frac{\text{forearm}}{\text{foot}}$	_____	_____ : _____
$\frac{\text{wrist}}{\text{perimeter of base of thumb}}$	_____	_____ : _____
$\frac{\text{neck}}{\text{wrist}}$	_____	_____ : _____
$\frac{\text{waist}}{\text{neck}}$	_____	_____ : _____
$\frac{\text{height}}{\text{foot}}$	_____	_____ : _____

19

Lemonade Mix: Teaching Notes

Francine Cabral Roy

GRADE RANGE: 5–9

MATHEMATICAL TOPICS: ratio, proportion, linear functions, tables, graphs

Discussion of the Mathematics

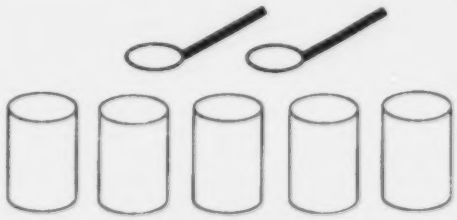
The important ideas are the connections among various aspects of ratio and proportion; for example, connecting the ratios embedded in the table with the slopes of the two line graphs, connecting the ratios of mix to water for different numbers of pitchers to make proportions; connecting slope to visual characteristics (e.g., position and steepness) of the graphs.

Implementation

This activity builds an understanding of proportional reasoning through the use of a physical context rather than mathematical procedures. Once understanding is established, more sophisticated procedures can be discussed. For example, after students solve problem 2, the teacher could help them compare additive strategies with multiplicative strategies. A multiplicative strategy serves as a quantitative underpinning for slope and can help students create closed algebraic representations of linear functions.

Students' Typical Responses

Students have the most difficulty with missing-value proportion problems (e.g., problem 4) that cannot be solved by equivalent fractions. There are several strategies that you might suggest to help students think about these problems: estimating, drawing a diagram, and guess and check. For example, students can draw a diagram representing 2 tablespoons (tbsp.) of mix and 5 cups of water and then estimate the solution (e.g., Is it less than 1? Less than 1/2?). Using the diagram, they can divide the 2 tbsp. of mix into equal portions and fit them into the 5 cups of water.



After guessing and checking, most students can divide each tbsp. into fifths and then distribute each fifth into one of the cups. Each cup of water (i.e., the unit rate) needs two fifths of a tbsp. of mix.

Additional Information

The activity relies heavily on problem solving, so you may need to encourage students to persevere. If students are having difficulty, focus their attention on the physical context and encourage them to estimate. In these problems, students' thinking and reasoning are valued over right or wrong answers.

Credit

Lappan, Glenda, James T. Fey, William M. Fitzgerald, Susan N. Friel, and Elizabeth Difanis Phillips. *Comparing and Scaling: Ratio, Proportion, and Percent.* Menlo Park, Calif.: Dale Seymour Publications, 1998.

Name _____ Date _____

Lemonade Mix

Below are two recipes for making lemonade. Recipe 1 calls for 2 tablespoons (tbsp.) of mix for each 5 cups of water. Recipe 2 calls for 4 tbsp. of mix for every 7 cups of water. Complete the table to determine the amount of mix and water for the given numbers of pitchers.

	RECIPE 1		RECIPE 2	
Pitchers	Lemonade Mix (tbsp.)	Water (Cups)	Lemonade Mix (tbsp.)	Water (Cups)
1	2	5	4	7
2				
3				
4				
5				
6				
7				
8				
9				
10				

1. Which recipe tastes more "lemony"? Explain at least two ways the table can help you choose the more lemony recipe.

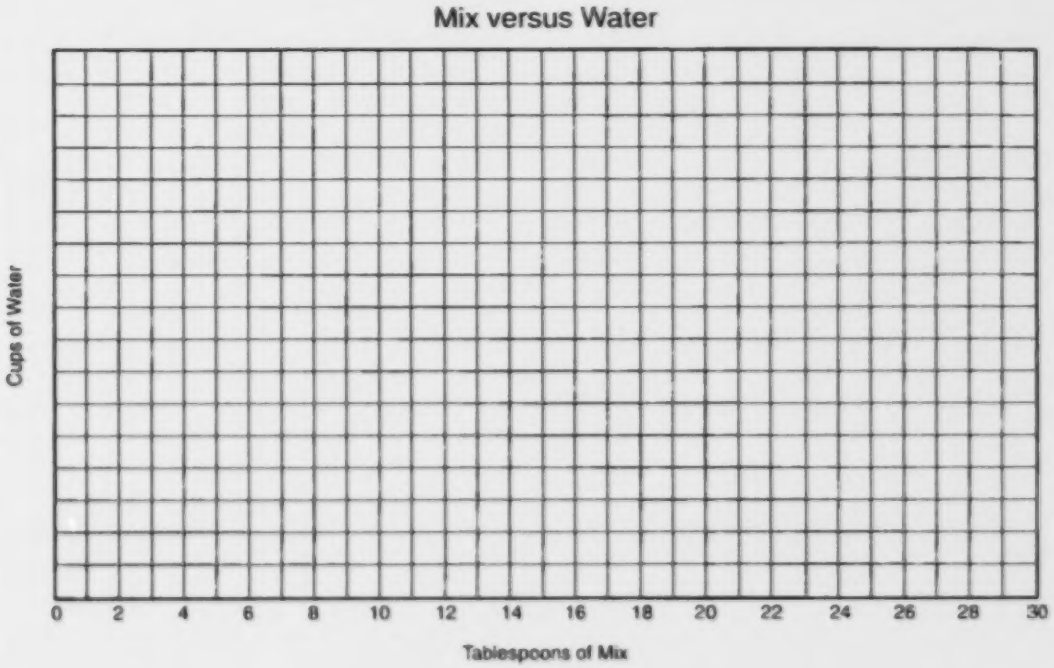
2. For Recipe 1, how many tbsp. of mix would you need to make 100 cups of lemonade? For 1000 cups of lemonade? Explain how you got your answers.

3. Suppose that you only had 1 tbsp. of mix. How many cups of lemonade could you make using Recipe 1? Using Recipe 2? Explain how you got your answers.

4. Suppose you want to make only one cup of lemonade. How much mix would you need using Recipe 1?

Using Recipe 2? Explain how you got your answers.

On the grid below draw line graphs for the amounts of mix and water for Recipe 1 and Recipe 2.



5. How are the graphs similar? How are they different? Why is one graph "higher" than the other? Why is one graph "steeper" than the other?

6. As the amount of mix increases, do the graphs seem to be further apart or closer together? Explain why this is happening.

7. How can you use the comparison of the two graphs to determine which recipe is more lemony?

20

Centimeters and Inches–A Different Look at Proportional Reasoning: Teaching Notes

Suzanne Levin Weinberg

GRADE RANGE: 6–10

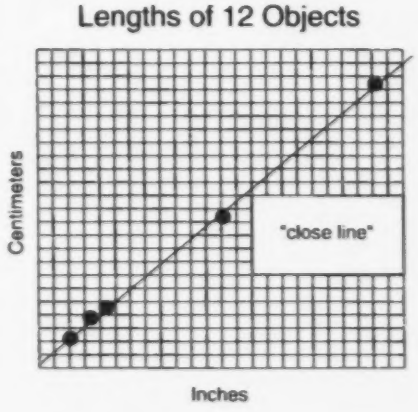
MATHEMATICAL TOPICS: proportion, ratio, equivalent fractions, measurement, graphing

MATERIALS: ruler or measuring stick with both inches and centimeters, calculator

Discussion of the Mathematics, with Answers and Solutions

This activity helps students connect the concepts of constant rate and measurement (2.54 cm per in.) to the idea of proportionality and slope. Many students solve proportional-reasoning problems by cross-multiplying, which masks the concept of unit ratio (or rate). This activity is not designed to reinforce the measurement conversion formulas. Emphasize the definition of a proportion as "a statement of equal ratios (or rates)," and discuss how writing equivalent fractions and decimals connects with the definition. Five sample measurements appear below.

Name of Object	Length (in.)	Length (cm)	Fraction cm/in.	Decimal cm 4 in.
Height of the desk	30	75	5/2	2.5
Length of pen	6	15	5/2	2.5
Height of cabinet	54.5	135	270/109	2.47
Width of paper	8.5	21.5	43/17	2.52
Length of paper	11	28	28/11	2.55



Answers

1. Yes, the lengths are in proportion.
2. The ratios of the length in centimeters to the length in inches, when the ratios are expressed in fractions or decimals, are the same for all the objects, if measurement error is considered.
3. The line should go through the origin because 0 inches corresponds to 0 centimeters.

4. Answers should be close to $y = 2.54 \cdot x$. 2.54 is the slope, showing that each inch corresponds to 2.54 centimeters in length. The decimal for each object in the chart is ⊕2.54.
5. This is another example of linear proportional reasoning, and so the points would approximate the line $y = 1.2 \cdot x$, when centimeters are plotted along the x-axis.. The slope is the constant ratio of 9 miles ÷ 5 centimeters, or 1.2 miles/centimeter.

Implementation

It is essential that students actually measure the objects with rulers, tape measures, or measuring sticks, rather than using conversion formulas. Remind students that measurements are not exact. The fractions in the chart should be approximately equal and the decimals should be approximately equal, which illustrates why this is a proportional situation. Be sure that students know how to plot ordered pairs for each object (length in inches, length in centimeters) on a grid. The ordered pairs yield a scatter plot that approximates the equation of the line $y = 2.54 \cdot x$. As an extension, graphing calculators can be used to plot the points and determine a line of best fit.

Questions to Ask Students

If one of your fractions or decimals is unusual, why did that happen? (A measurement error occurred.) What would happen if we put inches on the y-axis? (The slope of the line would be 1 inch/2.54 cm.) How do you know if two fractions are approximately equivalent? (The decimal equivalents are close.) How do you decide if two decimals are approximately equal? (Sample: They are within .02 or .05 of each other.)

Name _____ Date _____

Centimeters and Inches–a Different Look at Proportional Reasoning

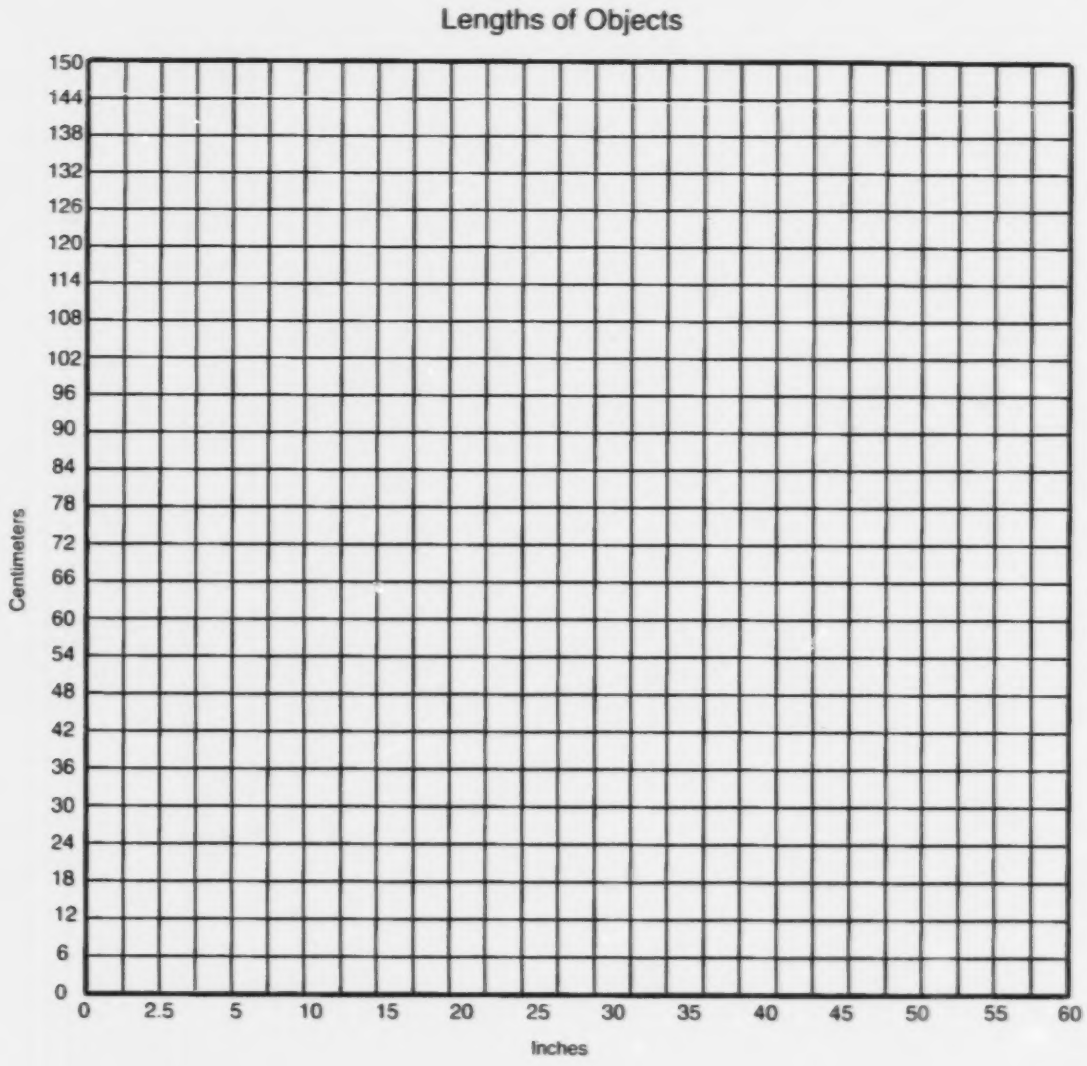
With a partner, measure the lengths of 12 objects in this classroom. The objects must have length between 5 inches and 60 inches. Measure each object in both inches and centimeters, and record the measurements in the chart below.

Name of Object	Length (cm)	Length (in.)	Fraction (cm/in.)	Decimal (cm ÷ in.)

1. Are the lengths (in centimeters and in inches) of the objects in proportion? Explain.

2. How do the data from the table help you decide whether the lengths (in centimeters and in inches) of the objects are in proportion?

The 12 objects you measured can be represented by ordered pairs of the form (length in inches, length in centimeters). Plot the ordered pairs for each object on the grid below.



3. Draw a line that is "close" to the points on the graph. Does the line go through the origin? Explain.

4. Write the equation for this line: $y =$ ____ $\cdot x$. What is the slope of the line? How does this value relate to the measurements you made?

5. Consider the situation: "On a certain map that is drawn to scale, 5 centimeters represents 9 miles." If you made a table and plotted points (centimeters on map, miles in the real world) that correspond to this situation, will the points form a line? Explain.

21

Conversion of Measurements–Unit Analysis: Teaching Notes

Elizabeth B. Appelbaum

GRADE RANGE: 5–12

MATHEMATICAL TOPICS: multiplication and division with fractions and decimals, measure conversion (dimensional analysis)

Discussion of the Mathematics, with Answers and Solutions

One of the goals is to help students keep track of units of measure. Here are examples: Driving at 45 miles per hour, how long will it take to go 200 miles? Multiply 200 miles by the "conversion fraction," 1 hr/45 mi. The units of hours are in the numerator. The units of miles are in the denominator. Names of units are cancelled as if they were numbers. Since the speed is 45 mph, the fraction, 1 hr/45 mi, can be considered equivalent to 1 for this problem.

$$200 \text{ mi} \times \frac{1 \text{ hr}}{45 \text{ mi}} = \frac{40}{9} \text{ hr} = 4\frac{4}{9} \text{ hr}$$

The fractional part can be converted to minutes, using the conversion fraction, 60 min/1 hr:

$$\frac{4}{9} \times \frac{60 \text{ min}}{1 \text{ hr}} = \frac{80}{3} \approx 27 \text{ min}$$

Cancelling numbers is easier before rather than after multiplication. If the calculation is done with a calculator, the result is 4.444444444 hr. To change the fraction of an hour to minutes, press –, 4, =, •, 60, =. The total time is about 4 hr, 27 min.

Some maps have "unit-free" scales. Suppose a map has scale 1:50,000. Convert 6 3/8 inches (map) to miles (ground):

$$6\frac{3}{8} \text{ in. (map)} \times \frac{50000 \text{ in. (ground)}}{1 \text{ in. (map)}} \times$$

$$\frac{1 \text{ ft}}{12 \text{ in.}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \quad 5.0 \text{ mi (ground)}$$

Answers

1. 84 m
2. 42.2 mi
3. 2.1 hr
4a. 1.8 hr
4b. 1 hr, 50 min
5. 1.4 mi
6. 1/6 hr, 1.2 hr
7. 4 1/4 ft, 4.3 ft
8. 66 in, 66 in
9. 151 2/3 mi, 151.7 mi
10. 1 37/40 km, 1.9 km
11. 89 71/80 mi, 89.9 mi
12. 7 9/11 hr, 7.8 hr
13. 7 hr, 49 min; 7 hr, 49 min
14. 4 1289/1584 mi, 4.8 mi
15. 4.48 km, 4.5 km

Name _____ Date _____

Conversion of Measurements–Unit Analysis

For each exercise, show the units of measure in your answer.

1. Suppose your average pace is 0.7 meters. (Pace is the length of one step while walking.) If you walk 120 paces, what is the distance in meters?

2. The scale on a map shows that 1 inch on the map is equivalent to 13.5 miles on the ground. If you measure a map distance of 3 1/8 inches, what is the ground distance? Round the answer to one decimal place.

3. Convert 125 minutes to hours. Write your answer as a decimal, correct to one decimal place.

4. Suppose campers hike a distance of 7.3 km at an average speed of 4 km per hour. How long does the hike last?

   a. Answer in hours, as a decimal, rounded to one decimal place.

   b. Answer in hours and minutes, correct to the closest minute.

5. A map has a unit-free scale, 1:25,000; that is, 1 inch on the map represents 25,000 inches on the ground. If the map distance between two points is 3 7/16 inches, find the ground distance in miles. Round the answer to one decimal place.

6. For each problem, write an expression that shows how to calculate the answer. Then do the calculation. If you calculate by hand, simplify the fractions. If you use a calculator, answer as a decimal, rounded to one decimal place.

	From	To	Expression	Answer
6	70 min	hours		
7	51 min	ft		
8	5 1/2 ft	in		
9	2 hr, 20 min at 65 mph	mi		
10	7.7 km at 4 km/hr	hr		
11	6 3/8 in. on a map, scale: 1 in. = 14.1 mi	mi on the ground		
12	430 mi at 55 mph	hr		
13	430 mi at 55 mph	hi and min, closest min		
14	1 5/8 in. on a map, scale: 1: 40,000	mi on the ground		
15	11.2 cm on a mpa, scale: 1:40, 000	km on the ground		

Excellent resource for mathematics educators! For the first time ever, an NCTM yearbook with a supplementary booklet that provides activities for teachers to use in their classrooms. A two-in-one package filled with information and ready-to-use materials.

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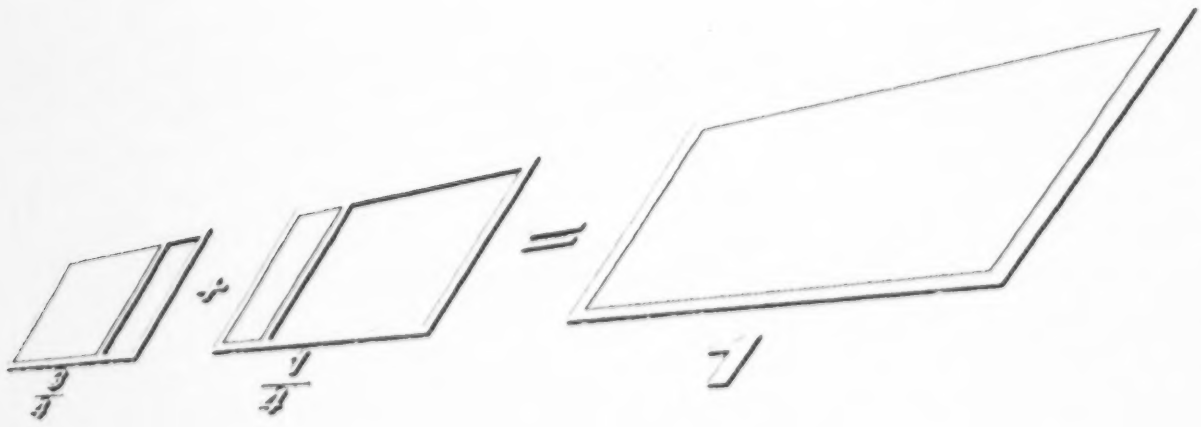

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